1994 - AB1

Let f be the function given by $f(x) = 3x^4 + x^3 - 21x^2$.

(a) Write an equation of the line tangent to the graph of f at the point $(2, -28)$.

(b) Find the absolute minimum value of f. Show the analysis that leads to your conclusion.

(c) Find the x-coordinate of each point of inflection on the graph of f. Show the analysis that leads to your conclusion.

1994 - AB5, BC2

Let R be the region enclosed by the graphs of $y = e^x$, $y = x$, and the lines $x = 0$ and $x = 4$.

(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the <u>x-axis</u>.

(c) Set up, but <u>do not integrate</u>, an integral expression in terms of a single variable for the volume of the solid generated when R is revolved about the <u>y-axis</u>.

1994 - AB3

Consider the curve defined by $x^2 + xy + y^2 = 27$.

(a) Write an expression for the slope of the curve at any point (x, y).

(b) Determine whether the lines tangent to the curve at the x-intercepts of the curve are parallel. Show the analysis that leads to your conclusion.

(c) Find the points on the curve where the lines tangent to the curve are vertical.

1994 - AB4

A particle moves along the x-axis so that at any time $t > 0$ its velocity is given by $v(t) = t \ln t - t$. At time $t = 1$, the position of the particle is $x(1) = 6$.

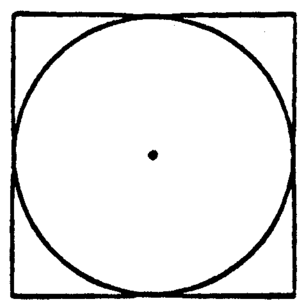
(a) Write an expression for the acceleration of the particle.

(b) For what values of t is the particle moving to the right?

(c) What is the minimum velocity of the particle? Show the analysis that leads to your conclusion.

(d) Write an expression for the position $x(t)$ of the particle.

1994 - AB2, BC1



A circle is inscribed in a square as shown in the figure above. The circumference of the circle is increasing at a constant rate of 6 inches per second. As the circle expands, the square expands to maintain the condition of tangency. (Note: A circle with radius r has circumference $C = 2\pi r$ and area $A = \pi r^2$.)

(a) Find the rate at which the perimeter of the square is increasing. Indicate units of measure.

(b) At the instant when the area of the circle is 25π square inches, find the rate of increase in the area enclosed between the circle and the square. Indicate units of measure.

1994 - AB6

Let $F(x) = \int_0^x \sin(t^2)\,dt$ for $0 \le x \le 3$.

(a) Use the trapezoidal rule with four equal subdivisions of the closed interval $[0, 1]$ to approximate $F(1)$.

(b) On what intervals is F increasing?

(c) If the average rate of change of F on the closed interval $[1, 3]$ is k, find $\int_1^3 \sin(t^2)\,dt$ in terms of k

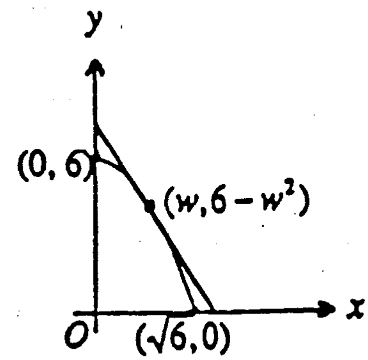
1994 – BC 3

A particle moves along the graph of $y = \cos x$ so that the x-component of acceleration is always 2. At time $t = 0$, the particle is at the point $(\pi, -1)$ and the velocity vector of the particle is $(0, 0)$.

(a) Find the x- and y- coordinates of the position of the particle in terms of t.

(b) Find the speed of the particle when its position is $(4, \cos 4)$.

1994 – BC 4



Note: Figure not to scale.

Let $f(x) = 6 - x^2$. For $0 < w < \sqrt{6}$, let $A(w)$ be the area of the triangle formed by the coordinate axes and the line tangent to the graph of f at the point $(w,\ 6 - w^2)$. (See figure above.)

(a) Find $A(1)$.

(b) For what value of w is $A(w)$ a minimum?

1994 – BC 5

Let f be the function given by $f(x) = e^{-2x^2}$.

(a) Find the first four nonzero terms and the general term of the power series for $f(x)$ about $x = 0$.

(b) Find the interval of convergence of the power series for $f(x)$ about $x = 0$. Show the analysis that leads to your conclusion.

(c) Let g be the function given by the sum of the first four nonzero terms of the power series for $f(x)$ about $x = 0$. Show that $|f(x) - g(x)| < 0.02$ for $-0.6 \le x \le 0.6$.

1994 – BC 6

Let f and g be functions that are differentiable for all real numbers x and that have the following properties.

(i) $f'(x) = f(x) - g(x)$

(ii) $g'(x) = g(x) - f(x)$

(iii) $f(0) = 5$

(iv) $g(0) = 1$

(a) Prove that $f(x) + g(x) = 6$ for all x.

(b) Find $f(x)$ and $g(x)$. Show your work.